The Behavior of Consumers

Here's a test of your prediction skills: A man goes into a restaurant. What does he order?

If your answer is "not enough information," you've passed the test. In fact, you're missing two different kinds of information here. First, you don't know what the man likes to eat. Second, you don't know what's available on the menu. To make a prediction, you'd need to know something about both the man's *preferences* and his *opportunities.*

Here's a harder test: A shopper goes into a grocery store. What does she buy? This is even harder than the man-in-the-restaurant question, because he's choosing one item off a menu, whereas the shopper is choosing an entire *basket* of items. But the same principle applies: To make predictions, you must know something both about preferences (how many lamb chops would the shopper *be willing* to sacrifice in order to have a cherry pie?) and opportunities (how many would the shopper *have to* sacrifice in order to have a cherry pie?). The "opportunities" question breaks down into two subquestions: What are the prices on the items, and how much income does the shopper have?

In Section 3.1, we'll think about preferences, and in Section 3.2 we'll think about opportunities. Once we put all that together, we can start making predictions.

3.1 Tastes

The Latin proverb "*De gustibus non est disputandum*" can be translated as "There's no accounting for tastes." Some people like antique wooden furniture; others prefer brass. You are likely to get a variety of answers if you ask different friends whether they would prefer to live in a world without Bach or in a world without clean sheets.

Economists accept the wisdom of the proverb and make no attempt to account for tastes. Why people prefer the things that they do is an interesting topic, but it is not one that we will explore. We take people's tastes as given and see what can be said about them.

Indifference Curves

Imagine a consumer named Beth who lives in a world with only two goods: eggs and root beer. You might imagine asking Beth which of these two goods she likes better. But

although this question sounds sensible at first, it really isn't—for several reasons. First, the answer is likely to depend on the quantities of eggs and root beer being compared. Second, the answer might depend on how many eggs and root beers she's already got in her refrigerator. The question is open to several interpretations: Are we asking which good Beth would least like to do without altogether, or are we asking which she would rather receive for her birthday?

Here is a better question: We can ask Beth whether she'd rather consume a basket of 3 eggs and 5 root beers or a basket with 4 eggs and 2 root beers. In principle, we could discover the answer by taking away all of Beth's possessions and then offering her a choice between the two baskets. A question makes sense when some (possibly imaginary) experiment is capable of revealing the answer.

Of course, there are many possible baskets besides the ones we've described. We can display all of them simultaneously on a graph, as in Exhibit 3.1. Each point on that graph represents a basket containing a certain number of eggs and a certain number of root beers. For example, point *A* represents a basket with 3 eggs and 5 root beers—the first of the 2 baskets we offered to Beth.

Exercise 3.1 Describe the baskets represented by points *B, C,* and *D.* Which represents the second basket of our imaginary experiment?


Goods

Items of which the consumer would prefer to have more rather than less.

What can we say about Beth's preferences among these baskets? Compare basket *A* to basket *B,* for example. Which would she prefer to own? Basket *B* contains more eggs than basket *A* (4 units instead of 3) and also more root beers (7 instead of 5). *If we assume that eggs and root beer are both* **goods**—items that Beth would prefer to have more of whenever she can—then the choice is unambiguous. Basket *B* is better than basket *A.*

EXHIBIT 3.1   **Basket of Goods**
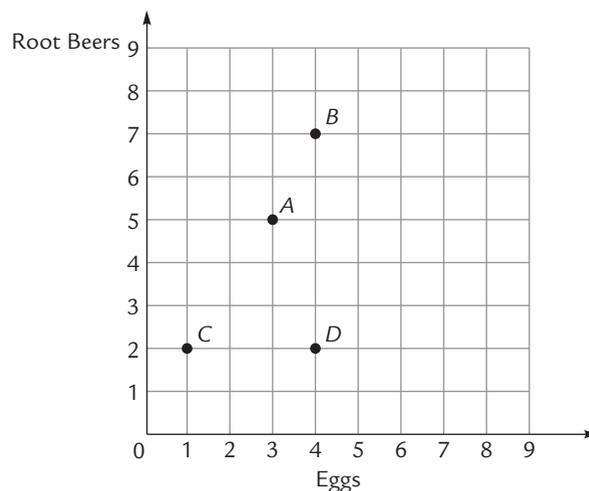
Each point on the graph represents a basket containing a certain number of eggs and a certain number of root beers. For example, point *A* corresponds to 3 eggs and 5 root beers.
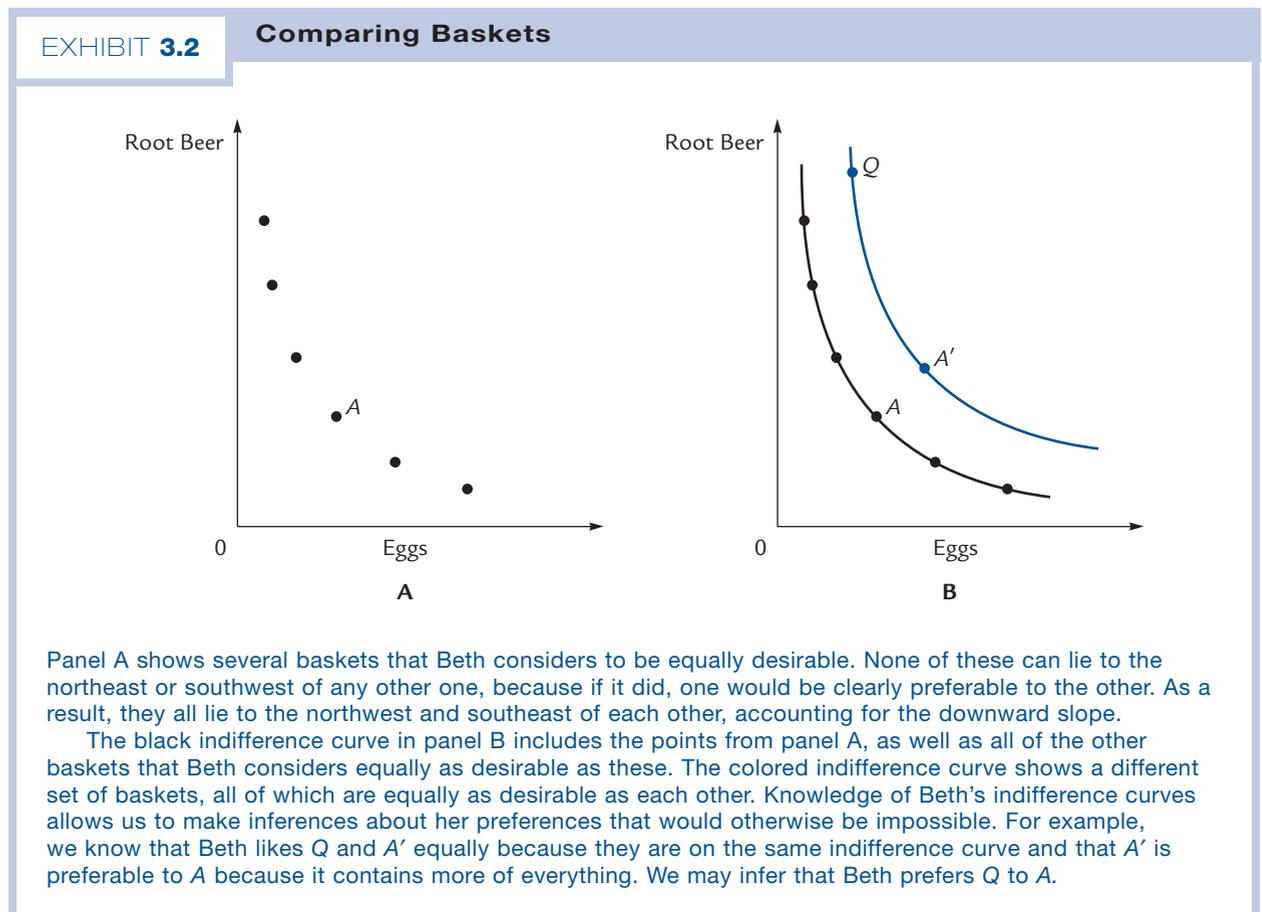

Exercise 3.2 Which is preferable, basket *A* or basket *C*? How do you know?

When it comes to comparing basket *A* with basket *D*, the choice is less clear. Basket *D* has more eggs (4 units versus 3) but less root beer (2 versus 5). Which will Beth prefer? At this point we cannot possibly say. She might like *A* better than *D*, or *D* better than *A*. It is also possible (though not necessary or even likely) that she would happen to like them both equally.

Now consider this question: Where should we look to find the baskets that Beth likes exactly as much as *A*? They can't be to the "northeast" of *A* (like *B*) because the baskets there are all preferred to *A*. They can't be to the "southwest" of *A* (like *C*) because *A* is preferred to all of those baskets. They must all be to either the "northwest" or the "southeast" of *A* (like *D*). This doesn't mean that *D* is necessarily one of them, just that they lie in the same general direction from *A* that *D* does.

If we draw in a few of the baskets that Beth likes just as well as she likes *A*, they might look like the points shown in panel A of Exhibit 3.2. Because each of these baskets is *exactly as good* as *A*, they must all be *exactly as good* as each other. This means that each one must lie either to the northwest or to the southeast of each other one, which accounts for the downward slope that is apparent in the picture.

The baskets shown in panel A of Exhibit 3.2 are only a few of those that Beth likes just as well as *A*. There are many other such baskets as well. The collection of all such

EXHIBIT 3.2 **Comparing Baskets**

Panel A shows several baskets that Beth considers to be equally desirable. None of these can lie to the northeast or southwest of any other one, because if it did, one would be clearly preferable to the other. As a result, they all lie to the northwest and southeast of each other, accounting for the downward slope.

The black indifference curve in panel B includes the points from panel A, as well as all of the other baskets that Beth considers equally as desirable as these. The colored indifference curve shows a different set of baskets, all of which are equally as desirable as each other. Knowledge of Beth's indifference curves allows us to make inferences about her preferences that would otherwise be impossible. For example, we know that Beth likes *Q* and *A′* equally because they are on the same indifference curve and that *A′* is preferable to *A* because it contains more of everything. We may infer that Beth prefers *Q* to *A*.

baskets forms a curve, shown in black in panel B of the exhibit. From our discussion in the preceding paragraph, we know that the curve will be downward sloping. Because Beth is indifferent between any two points on this curve, it is called an **indifference curve.**

Indifference curve

A collection of baskets, all of which the consumer considers equally desirable.

There is nothing special about basket *A*. We could as easily have begun with a different basket, such as *A′* in panel B of Exhibit 3.2. That panel depicts both the indifference curve through *A* (in black) and the indifference curve through *A′* (in color).

The indifference curves do not have to have the same shape, but they do both have to slope downward.

If we know a consumer's indifference curves, we can make inferences that would not be possible otherwise. Try comparing basket *A* to basket *Q* in panel B of Exhibit 3.2. Basket *A* has more eggs than basket *Q*, but basket *Q* has more root beer than basket *A*. Without more information, we cannot say which one Beth will prefer. But the indifference curves provide that additional information. We know that Beth likes *Q* and *A′* equally, because they are on the same indifference curve. We know that she likes *A′* better than she likes *A*, because it is to the northeast of *A* and therefore contains more of everything than *A* does. We may infer that she likes *Q* better than she likes *A*.

In general, a basket is preferable to another precisely when it is on a higher indifference curve, where *higher* means "above and to the right."

Relationships among Indifference Curves

Of course, Beth has more than two indifference curves. Indeed, we can draw an indifference curve through any point we choose to start with. Because of this, *the indifference curves fill the entire plane.* (More precisely, they fill the entire quadrant of the plane in which both coordinates are positive.)

An important feature of indifference curves is that *indifference curves never cross.* To understand why this must be true, imagine a consumer with two indifference curves that cross, as in Exhibit 3.3.

From the fact that baskets *P* and *Q* are on the same (black) indifference curve, we know that the consumer likes these baskets equally well. From the fact that baskets *R* and *Q* are on the same (brown) indifference curve, we know that he also likes these equally well. Putting these facts together, we conclude that he likes *P* and *R* equally well. But this is impossible, because *R* is to the northeast of *P* and therefore contains more of both goods. In other words, if indifference curves cross, impossible things will happen. We conclude that indifference curves don't cross.
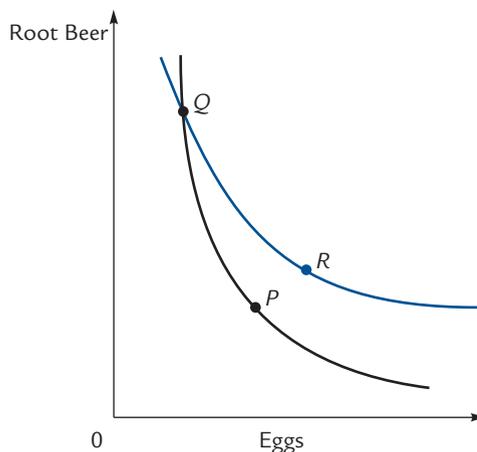
Marginal Values

We have said that indifference curves slope downward, but we haven't yet said anything about how steep the slope is. In this section, we will interpret the slope of the indifference curve. The first step is to understand how indifference curves can tell us whether certain trades are desirable.

Desirable and Undesirable Trades

Suppose you have 7 eggs and 2 root beers; this basket is represented by point *C* in Exhibit 3.4. Your friend Jeremy offers to trade you 2 root beers for an egg. If you accept his offer, you'll end up at point *P*. (That is, you'll give Jeremy an egg, leaving you with 6 eggs, and he'll give you 2 root beers, leaving you with 4 root beers. Point *P* illustrates your new basket.)

EXHIBIT **3.3**	**Indifference Curves Never Cross**

Root Beer graph with Eggs on horizontal axis, showing points Q, R, and P on two crossing indifference curves.

Crossing indifference curves, such as those shown in the graph, cannot occur. The consumer likes *P* and *Q* equally well because they are both on the same (black) indifference curve. He also likes *R* and *Q* equally well because they are both on the same (colored) indifference curve. We may infer that he likes *P* and *R* equally well, which we know to be false (in fact, *R* is preferred to *P*). Thus, the graph cannot be correct.

Will you accept Jeremy's offer? It depends on your preferences. Suppose, for example, that you have the indifference curve shown in Exhibit 3.4. Then you will *not* accept Jeremy's offer, because—according to your preferences—point *P* is inferior to point *C*.

In other words, when Jeremy says, "I'll give you 2 root beers for an egg," you'll say, "No thanks; I'd rather keep the egg." In ordinary language, we'd say that your seventh egg is worth more to you than 2 root beers.

Suppose Jeremy tries again, by offering you 4 root beers for an egg instead of 2 root beers. Now do you accept the trade? If you do, you'll end up at point *R,* above your original indifference curve. This trade is desirable; it makes you happier; you got more for your egg than you thought it was worth.

Exercise 3.3 Explain why Jeremy's new offer brings you to point *R*.

Finally, what if Jeremy had offered you exactly 3 root beers for your seventh egg? This brings you to point *Q,* which is exactly as desirable as your original point *C.* That is, trading an egg for 3 root beers makes you neither better nor worse off than you were to begin with. This makes it reasonable to say that your seventh egg is *worth* exactly 3 root beers (to you). We say that (to you) the **marginal value** of an egg is 3 root beers.

In general, the marginal value that you place on good *X* (in terms of good *Y*) is defined to be the number of *Y*s for which you'd be just willing to trade one *X*.[1] (The adjective *marginal* refers to the fact that you are trading just *one X*.)

Given a consumer's initial basket and the indifference curve through that basket, you can always compute the marginal value of the horizontal good by traveling leftward

Marginal value of X in terms of Y

The number of *Y*s for which the consumer would be just willing to trade one *X*.

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[1] In many textbooks, the marginal value is called the *marginal rate of substitution* or *MRS.* Unfortunately, there is quite a bit of confusion associated with this term. The quantity that we've called the marginal value of *X* in terms of *Y* is sometimes called the marginal rate of substitution between *X* and *Y,* and sometimes called the marginal rate of substitution between *Y* and *X.* To avoid this confusion, we will stick with the term *marginal* value.

EXHIBIT **3.4**   **Marginal Value**
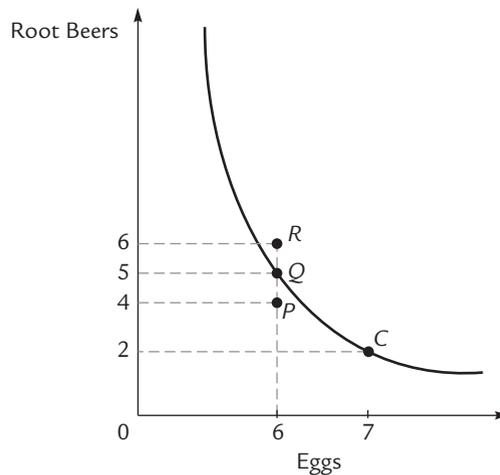
Suppose you start with basket *C.* If someone offers to trade you 2 root beers for an egg, you can move to basket *P,* which is worse; so you'll reject this trade. The minimum price you'd accept for an egg is 3 root beers, moving you to basket *Q.* Thus (to you), the marginal value of an egg is 3 root beers.

---

1 unit and then seeing how far upward you must travel to reach the indifference curve. In Exhibit 3.4, this means starting at point *C,* traveling leftward 1 egg (from 7 to 6), and then observing that you must travel upward 3 root beers (from 2 to 5); thus—as we have already said—the marginal value of an egg is 3 root beers.

Exercise 3.4 How can you use the indifference curve of Exhibit 3.4 to illustrate the marginal value of *root beers* in terms of *eggs?*

Marginal Value as a Slope

Exhibit 3.5 illustrates the indifference curves of two consumers, each starting with basket *C.* We can use these indifference curves to compute the marginal value of an egg to each consumer. For Jack, the marginal value of an egg is 6 root beers; for Jill, the marginal value of an egg is 1 root beer.


Exercise 3.5 Explain how to compute these marginal values from the graphs in Exhibit 3.5.

Now let's forget about marginal values for a moment and ask a purely geometric question: What is the *slope* of Jack's indifference curve at point *C?* By the slope of a curve we mean the slope of a line tangent to that curve. The tangent line at *C* is well approximated by the illustrated line through *C* and *D.* So we want to compute the slope of that line.

Recalling that the slope of a line is given by the *rise over the run,* we see that in this case the slope is $-6/1 = -6$. The numerator 6 is the vertical distance between points

EXHIBIT 3.5 **Marginal Value as a Slope**

A: Jack's indifference curve   **B: Jill's indifference curve**
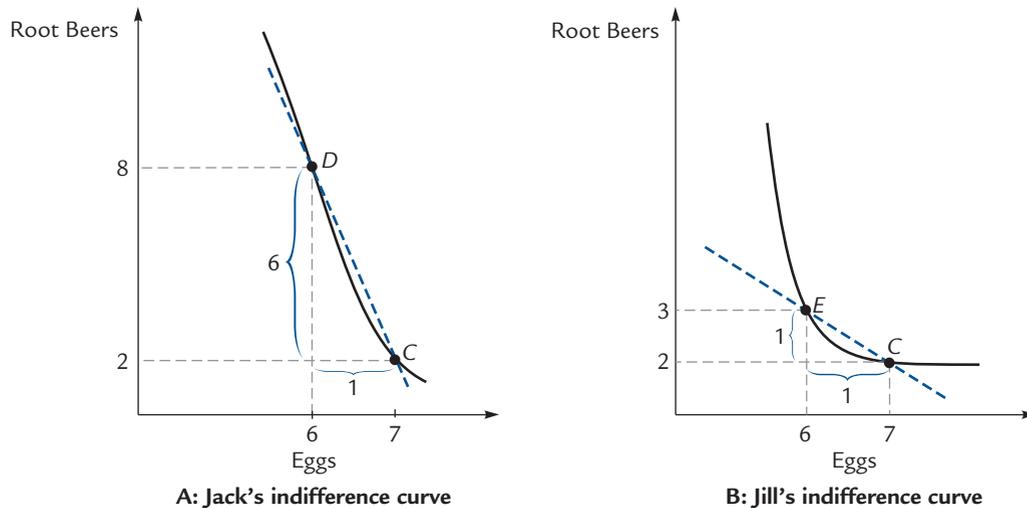
Jack and Jill each start with basket *C.* To Jack, the marginal value of an egg is 6 root beers. Thus, trading 1 egg for 6 root beers leaves him on the same indifference curve; in other words, his indifference curve goes through point *D* and so has slope –6. To Jill, the marginal value of an egg is 1 root beer, so her indifference curve goes through point *E* and has slope –1. In general, the marginal value of an egg is equal to the absolute value of the slope of the indifference curve.

C and *D,* the denominator 1 is the horizontal distance, and there is a minus sign because the curve is downward sloping. The absolute value of this slope is 6 (or, more precisely, 6 root beers per egg). Recall that according to Jack, this is exactly the marginal value of an egg.

Likewise, in panel B the line through *C* and *E* has a slope with absolute value 1, which according to Jill is the marginal value of an egg.

It is no coincidence that these slopes are equal to the corresponding marginal values. In panel A, for example, we compute the marginal value of an egg as the vertical distance from *D* to *C* (that is, 6), while we compute the absolute value of the slope as that same vertical distance divided by the horizontal distance, which is 1. But dividing by 1 leaves the number 6 unchanged.

In general, then, **for a consumer with basket *C,* the marginal value of an egg is equal to the slope of the indifference curve at point *C.* ** Consequently, **the steeper the indifference curve, the greater the marginal value of an egg**.

The Shape of Indifference Curves

A starving person with a refrigerator full of root beer is likely to value an egg more highly (in terms of root beer) than a thirsty person with a refrigerator full of eggs. Because marginal value is reflected by the slopes of indifference curves, we can translate this statement into geometry: As a general rule, we expect indifference curves to be steep near baskets containing few eggs and many root beers and to be shallow near baskets containing many eggs and few root beers.

A   **B**
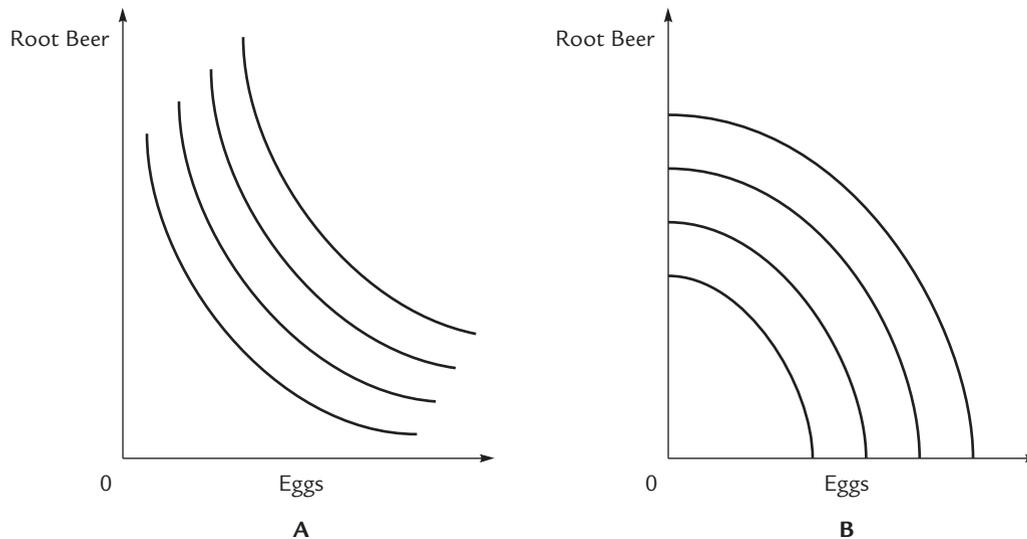
The indifference curves in panel A are convex (bowed in toward the origin), indicating that when the consumer has few eggs and many root beers (in the "northwest" part of the diagram), she places a high marginal value on eggs—that is, you'd have to offer her a lot of root beer to get her to part with an egg. We assume that indifference curves have this shape, rather than the alternative shape illustrated in panel B.

Consider the two sets of indifference curves shown in Exhibit 3.6. Both sets slope downward. The first set slopes steeply in the area where baskets contain few eggs and many root beers (that is, in the "northwest" part of the figure) and shallowly in the area where baskets contain few root beers and many eggs. This consumer conforms to the general rule of the preceding paragraph.

Another consumer might have the indifference curves shown in panel B of Exhibit 3.6. This consumer values eggs highly when she has many eggs and few root beers, but values eggs much less when she has few eggs and many root beers. Such tastes are possible, but they seem unlikely.

Therefore, we will always assume that indifference curves are shaped like those in panel A rather than those in panel B. That is, we assume that indifference curves bow inward toward the origin. This property is expressed by saying that indifference curves are **convex.** At the end of Section 3.2 we will give another, independent justification for assuming convexity.

Convex

Bowed in toward the origin, like the curves in panel A of Exhibit 3.6.

Exercise 3.6 Under what circumstances do you expect the consumer to value additional root beers highly relative to additional eggs? Combine this answer with your answer to Exercise 3.4 to draw a conclusion about where the indifference curves should be steep and where they should be shallow. Does your conclusion give further support to our assumption that indifference curves are convex, or does it suggest a reason to doubt that assumption?

More on Indifference Curves

Properties of Indifference Curves: A Summary

Here are the fundamental facts about a given consumer's indifference curves:

Indifference curves slope downward, they fill the plane, they never cross, and they are convex.

A consumer's indifference curves between two goods encode everything that there is to say about the consumer's tastes regarding those goods. A different consumer is likely to have a different family of indifference curves (also satisfying the fundamental facts). This is just another way of saying that tastes may differ across individuals.

We have assumed that eggs and root beer are both *goods*—items you'd always prefer to have more of—and we've concluded that indifference curves are downward sloping and convex. A different assumption would lead to different conclusions. End-of-chapter problems 5 and 6 will lead you through the analysis when one or both of the goods is replaced by a *bad*—something you'd prefer to have less of. (In problems 3 and 4, you'll encounter other special circumstances in which the shapes of indifference curves can differ from what is pictured in Exhibit 3.6A.)

Dangerous
Curve

The Composite-Good Convention

In order to draw indifference curve diagrams, we must assume that there are only two goods in the world. This might appear to be a severe limitation, yet in fact it is not. In many applications we will want to concentrate our attention on a single good—say, eggs. In that case we divide the world into two classes of goods, namely, "eggs" and "things that are not eggs," otherwise known as "all other goods." This allows us to draw indifference curves between eggs (on the horizontal axis) and all other goods (on the vertical).

There remains the problem of units. What is a single unit of *all other goods?* The simplest solution to this problem is to measure all other goods in terms of their dollar value.

When we lump together all things that are not eggs and measure it in a single unit like dollars, we say that we are using the **composite-good convention.**

In the presence of the composite-good convention, the slope of an indifference curve is the marginal value of an egg in terms of other goods, with the other goods measured in dollars. Thus, it is the minimum number of dollars for which the consumer would be willing to trade an egg.

Composite-good convention

The lumping together of all goods but one into a single portmanteau good.

3.2 The Budget Line and the Consumer's Choice

To predict a consumer's behavior, we need to know two things. First, we need to know the consumer's tastes, which is the same thing as saying that we need to know his indifference curves. Second, we need to know the options available to the consumer. In other words, we need to know his budget.

The Budget Line

Continue to assume a world with two goods. Instead of calling them eggs and root beers, we're going to start calling them X and Y. You may continue to think of them as eggs and root beers if you wish. In order to determine which baskets our consumer can afford, we need to know three things: the price of X, the price of Y, and the consumer's income.

Rather than make up specific numbers, let's make up names for the three things we need to know:

$$P_X = \text{the price of } X \text{ in dollars}$$
$$P_Y = \text{the price } of \, Y \text{ in dollars}$$
$$I = \text{the consumer's income in dollars}$$

Now let's suppose that the consumer is considering the purchase of a particular basket. Suppose that the basket contains x units of X and y units of Y. (Keep in mind that the capital letters X and Y are the *names* of the goods and the small letters x and y are the *quantities*.) How much will it cost the consumer to acquire this basket? The x units of X at a price of P_X dollars apiece will cost $P_X \cdot x$ dollars. The y units of Y at a price of P_Y dollars apiece will cost $P_Y \cdot y$ dollars. The total price of the basket is

$$P_X \cdot x + P_Y \cdot y \text{ dollars}$$

Under what circumstances can the consumer afford to acquire this particular basket? Clearly, he can acquire it only if the price of the basket does not exceed his income. In other words, he can afford the basket precisely if

$$P_X \cdot x + P_Y \cdot y \leq I$$

In fact, we can say a little more. Let's take seriously our assumption that X and Y are the only goods in the world. (In view of the composite-good convention, this assumption is not as outrageous as it seems.) Then the consumer will have to spend his entire income on X and Y,[2] and must choose a basket that costs exactly I dollars. The consumer can have the basket in question precisely if

$$P_X \cdot x + P_Y \cdot y = I$$

Dangerous Curve

It is important to distinguish the meanings of the various symbols in this equation. P_X, P_Y, and I are particular, fixed numbers that the consumer faces. The letters x and y are variables that can represent the contents of any basket. As the consumer considers purchasing various baskets, the values of x and y change. For each basket he plugs the relevant values of x and y into the equation, and he asks if the equation is true. Asking "Does this basket make the equation true?" is exactly the same as asking "Can I afford to purchase this basket?"

Budget line

The set of all baskets that the consumer can afford, given prices and his or her income.

The line described by the equation $P_X \cdot x + P_Y \cdot y = I$ is a picture of all the baskets that the consumer can afford. It is called the consumer's **budget line.**

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[2] It is possible that the consumer would want to save some income, but in that case we would want to consider savings as another good. If we are using the composite-good convention, we can include savings along with "all other goods."

Another way to write the equation of the budget line (using some simple algebraic manipulations) is

$$y = -\frac{P_X}{P_Y} \cdot x + \frac{I}{P_Y}$$

If you remember that P_X, P_Y, and I are constants and that x and y are variables, you may recognize this as the equation of a line with slope $-P_X/P_Y$ and y-intercept I/P_Y. The points on that line are those that satisfy the equation and are therefore those that represent baskets that the consumer can buy. Exhibit 3.7 shows the budget line.

Here is an easy way to remember how to draw the budget line. If you were the consumer and you bought no Xs at all, how many Ys could you afford? Because your income is I and Ys sell at a price of P_Y apiece, the answer is I/P_Y. This means that the point $(0, I/P_Y)$ must be on the budget line. If you bought no Ys at all, how many Xs could you afford? The answer is I/P_X. This means that the point $(I/P_X, 0)$ must be on the budget line. The budget line must be the line connecting the points $(0, I/P_Y)$ and $(I/P_X, 0)$.

What if P_X, P_Y, and I were all to double simultaneously? This would have no effect on the ratios I/P_Y and I/P_X. It follows that a simultaneous doubling of all prices and income would have no effect on the budget line. This accords with our expectation that only relative prices matter.

The geometry of the budget line reflects everything there is to know about the opportunities facing the consumer. For example, the slope of the budget line is $-P_X/P_Y$, and the ratio P_X/P_Y is the relative price of X in terms of Y. Therefore, the budget line will be steep when X is expensive relative to Y, and it will be shallow when X is inexpensive relative to Y.

EXHIBIT **3.7**   **The Budget Line**
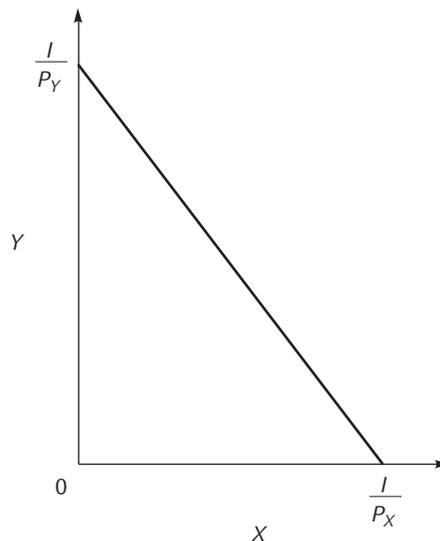
The consumer's budget line depicts the various baskets that he can afford with his income.

The Consumer's Choice

### The Geometry of the Consumer's Choice

The budget line conveys an entirely different kind of information than the indifference curves do. The indifference curves reflect the consumer's preferences without regard to what he can actually afford to buy. The budget line shows which baskets he can afford to buy (that is, it shows his opportunities) without regard to his preferences. To determine how the consumer will actually behave, we must combine these two kinds of information. To this end, we have drawn the indifference curves and the budget line on the same graph, as in Exhibit 3.8.

We now have enough information to determine which basket this consumer will choose. Look at the baskets pictured. Of these, *F* is on the highest indifference curve and the one that the consumer would most like to own. (There are also many baskets not pictured that the consumer would like even more than *F*.) Unfortunately, he can't afford basket *F*—it's outside the budget line. By contrast, point *E* is inside the budget line and would fail to exhaust his income; therefore, *E* is ruled out as well. The baskets that the consumer can acquire are the ones on his budget line. In Exhibit 3.8 these include *A, B, O, C,* and *D.*

Of these, he will choose the one on the highest possible indifference curve. It is clear from the picture that this choice is *O.* In fact, *O* is not just the best choice among the five baskets we have considered but the best choice of any basket on the budget line. From the picture, the following is clear:

| EXHIBIT **3.8** | **The Consumer's Optimum** |
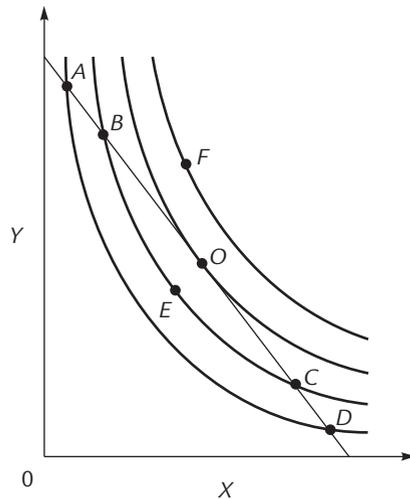
The consumer must choose one of the baskets that is on his budget line, such as *A, B, O, C,* or *D.* Of these, he will choose the one that is on the highest indifference curve, namely, *O.* Thus, the consumer is led to choose the basket at the point where his budget line is tangent to an indifference curve. This point is called the consumer's optimum.

At the consumer's optimum, the relative price of *X* in terms of *Y* (given by the slope of the budget line) and the marginal value of *X* in terms of *Y* (given by the slope of the tangent line to the indifference curve) are equal. The geometric reason for this is that the budget line is the tangent line to the indifference curve. The economic reason for it is that whenever the relative price is different from the marginal value, the consumer will continue to make exchanges until the two become equal.

The basket the consumer chooses will always be located where his budget line is tangent to one of his indifference curves.

This basket is called the consumer's **optimum.** Because there is only one such point, the budget line and the indifference curves give sufficient information for us to predict which basket the consumer will choose.

The Economics of the Consumer's Choice

We can analyze the consumer's problem from a different perspective and still reach the same conclusion about the location of his optimum.

Referring to Exhibit 3.8, suppose that the consumer owns basket *A*. How much *Y* would this consumer be willing to trade for an additional unit of *X?* The answer is given by the marginal value of a unit of *X* (in terms of *Y*), which is measured by the absolute value of the slope of his or her indifference curve at *A*.

How much *Y* would this consumer actually have to sacrifice in order to acquire an additional unit of *X?* The answer is given by the relative price of *X* in terms of *Y,* which is the ratio $P_X/P_Y,$ the absolute value of the slope of his budget line.

Of these two, which is greater, the marginal value or the relative price? At point *A* the indifference curve is steeper than the budget line. Consequently, the amount of *Y* that the consumer is *willing* to pay for a unit of *X* exceeds the amount of *Y* that he actually *has to* pay for a unit of *X.* In such a situation, buying a unit of *X* is an attractive proposition. The consumer will exchange *Y*s for *X*s at the going relative price, ending up with more *X* and less *Y* than he started with. This will bring him to a point like *B*.

Now the same reasoning applies again. At *B* it is still the case that the marginal value exceeds the relative price. The consumer will want to buy another unit of *X*, which will move him further down the budget line.

This process will continue until the consumer reaches point *O*. At that point the price that he is willing to pay for *X* and the price at which he is able to purchase *X* have become equal. There is no longer anything to be gained from additional trades.

A similar process occurs if the consumer starts out with basket *D*. Here the marginal value of *X* is less than the relative price of *X;* the consumer values his last unit of *X* at less than the number of *Y*s that can be exchanged for it in the marketplace. In this case, he will happily trade away his last unit of *X,* ending up with more *Y*s and fewer *X*s, at a point like *C*.

As long as the marginal value of *X* is less than the relative price of *X,* the consumer will trade *X*s for *Y*s. This process stops when the marginal value and the relative price become equal, at point *O*.

Whenever the marginal value of *X* exceeds the relative price of *X,* the consumer will want to buy *X*s, moving down the budget line. Whenever the marginal value is less than the relative price, the consumer will want to sell *X*s, moving up the budget line. The only point at which he can settle is *O,* where the marginal value and the relative price are exactly equal. Thus, the economic reasoning leads to the same conclusion as the geometric reasoning: Of the points available to the consumer, the optimum occurs where his budget line is tangent to one of his indifference curves.

Corner Solutions

There is an exception to the rule that the consumer's optimum always occurs at a tangency. This exception is illustrated in Exhibit 3.9. In this case there is no tangency for the consumer to choose.

Optimum (plural: optima)

The most preferred of the baskets on the budget line.

| EXHIBIT **3.9** | **A Corner Solution** |
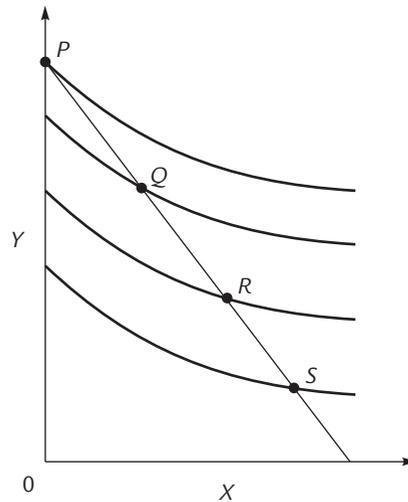
If the consumer's indifference curves look like those pictured, there is no tangency between his budget line and any of his indifference curves. Of all the points on the budget line, the consumer will choose the most desirable, namely, *P.* At any other point on the budget line the marginal value of *X* in terms of *Y* is less than the relative price, so the consumer can sell *X*s for more than they are worth to him and will continue to do so until he has sold all of his *X*s, ending up in the corner at *P.*

To predict the consumer's choice in this situation, we can use simple geometry. We know that the consumer must choose a basket on his budget line. Of all of these baskets, we can see from the picture that the one lying on the highest indifference curve is *P.* Therefore, the consumer chooses basket *P.*

Here is an alternative path to the same conclusion: Suppose the consumer begins with basket *S.* At this point his indifference curve is less steep than his budget line. To this consumer the marginal value of *X* in terms of *Y* is less than the relative price of *X* in terms of *Y.* The last unit of *X* is worth less to him than it will bring in the marketplace. Therefore, he trades *X* for *Y,* moving to a point like *R.* Now the same reasoning applies again, leading the consumer to move first to *Q* and then to *P.* The same reasoning would apply no matter what the original basket was.

Corner solution

An optimum occurring on one of the axes when there is no tangency between the budget line and an indifference curve.

The situation depicted in Exhibit 3.9 is called a **corner solution** because the consumer's optimum occurs in a corner of the diagram. As you can see from the picture, he consumes no *X* whatsoever and spends all of his income on *Y.*

More on the Shape of Indifference Curves

In Section 3.1 we justified the assumption that indifference curves are convex with an appeal to the idea of marginal value. Now we can give an additional reason for making this assumption.

Suppose a consumer has the indifference curves illustrated in Exhibit 3.10. Will this consumer choose to purchase the basket at point *O?* No! He can do better. Points *C* and *D* are both available to him (they are on his budget line), and they are on a higher indifference curve than *O.* And can he do better than *C* and *D?* Yes. Every movement

EXHIBIT **3.10**   **The Consumer's Choice with Nonconvex Indifference Curves**
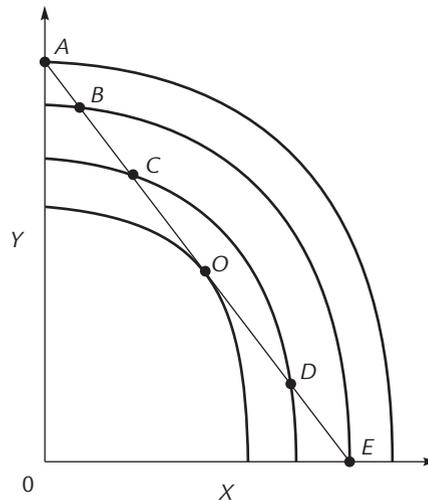
Nonconvex indifference curves always lead to a corner solution. The consumer pictured here will choose point *A,* which is on the highest possible indifference curve.

"outward" along the budget line, away from *O* and toward one of the axes, improves the consumer's welfare. For this reason he will always want to choose a basket on one of the axes—a corner solution. In this case he will choose basket *A*.

Exercise 3.7 Why does the consumer choose basket *A* rather than basket *E?* How would the budget line have to look for him to choose a point on the *X*-axis rather than the *Y*-axis?

Because this consumer always selects a corner solution, he consumes either zero units of *X* or zero units of *Y*. But goods that consumers choose to purchase none of are not very interesting from the viewpoint of economics. So now we have our additional reason for assuming that indifference curves are convex. They might not be—but in this case one of the goods in question would not be consumed at all, and we would prefer to turn our attention to goods that *are* consumed. Therefore, we usually confine our attention to convex indifference curves.

3.3 Applications of Indifference Curves

Now let's put our new tools to use. In this section we'll see several applications of indifference curve analysis.

Standards of Living

Economic conditions change all the time. Incomes go up and down, and so do prices. How do we tell which changes are good for the consumer and which are bad?

Sometimes it's easy. If your friend Harold's income goes up while prices remain unchanged, his life has certainly improved. If his income stays fixed while all prices rise, he's worse off than before. But what if some prices rise while others fall? Is that good or bad for Harold?

Sometimes there's not enough information to answer that question. Other times there is. Let's take an example: Harold consumes goods X and Y. Their prices are $P_X = \$3$ and $P_Y = \$4$. He chooses to buy 4 units of X and 2 of Y, exhausting his income of $20. Now the price of X rises to $4 while the price of Y falls to $2, and his income stays fixed at $20. Is Harold better or worse off than before?

To answer, start by drawing Harold's original budget line, marked *Original* in Exhibit 3.11. Given his income of $20 and given $P_X = \$3$, Harold can afford up to $6^2/_3$ Xs if he buys no Ys. Because $P_Y = \$4$, he can afford up to 5 Ys if he buys no Xs. Those calculations determine the two endpoints of his *Original* budget line. We are told that Harold chooses basket $O = (4, 2)$, so there must be an indifference curve tangent to the *Original* budget line at that point, as illustrated in the exhibit.

Now let's draw Harold's *New* budget line, after the prices change to $P_X = \$4$ and $P_Y = \$2$. The endpoints are at $X = 5$ and $Y = 10$, as shown in the exhibit. But knowing the endpoints is not enough to draw the *New* budget line accurately. We also have to think about whether it passes above, below, or through the point O.

To settle this question, ask whether Harold can afford basket O at the *New* prices. With $P_X = \$4$ and $P_Y = \$2$, basket O costs $(\$4 \times 4) + (\$2 \times 2) = \$20$, which is exactly Harold's income. So O must be on his *New* budget line, or, to put it another way, his *New* budget line must pass through point O. That's how we've drawn it in Exhibit 3.11.

Now let's find Harold's *New* optimum point—the point where his *New* budget line is tangent to an indifference curve. The first thing we can do is rule out point O. That's
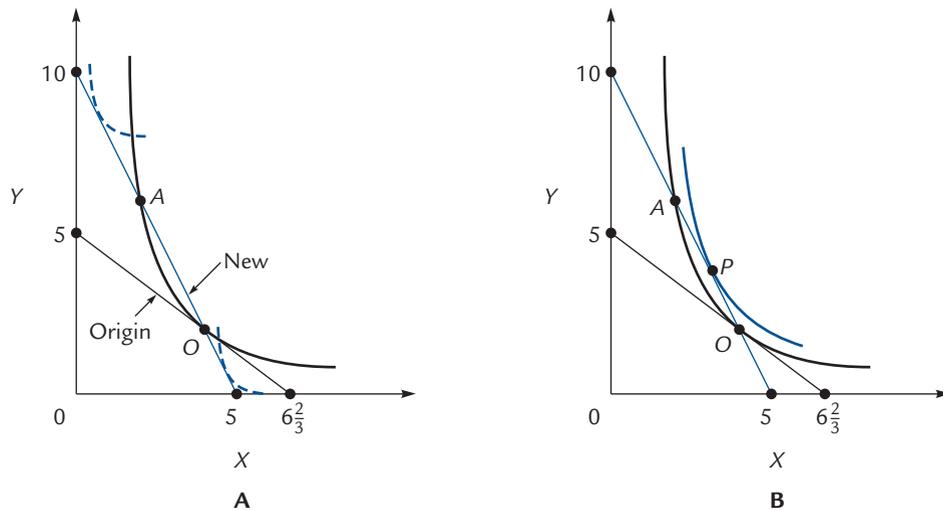

EXHIBIT 3.11   **Harold's Original and New Budget Lines**
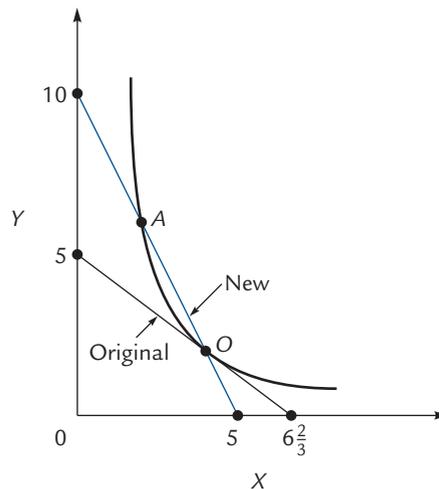
The graph shows Harold's *original* and *new* budget lines. We know that an indifference curve is tangent to the *original* line at the point O. We can calculate that the new budget line passes through the point O.

EXHIBIT 3.12 Finding the New Optimum

A

B

The dashed indifference curves in panel A cannot be correct, because they cross the indifference curve through *O.* The only correct way to draw an indifference curve tangent to the *new* budget line is with the tangency between *A* and *O,* at a point like *P,* as in panel B. The new indifference curve is then necessarily higher than the old one, so you are better off at the new optimum.

because *a smooth curve cannot be tangent to two different lines at the same point*—an important fact about geometry that will be useful to keep in mind.

So where is Harold's new optimum? Panel A of Exhibit 3.12 explores some possibilities. The two dashed curves are not possible, because either of them would have to cross the original indifference curve. That means Harold can't have an optimum in the region above and to the left of *A* or in the region below and to the right of *O.* Instead, his new optimum must lie between *A* and *O,* for example, at *P* in panel B. If you look at the panel, you'll see that *P* must lie on a higher indifference curve than *O.* Therefore, the price changes must have made Harold better off.

Price Indices

To measure how people are affected by price changes, the U.S. Department of Labor, through its Bureau of Labor Statistics, reports estimates of changes in the "cost of living," also called the "price level." Roughly, they do this by tracking the cost of a given basket over time. If the basket gets more expensive, they say that the cost of living has gone up (which suggests that people are worse off); if the basket gets cheaper, they say that the cost of living has gone down (which suggests that people are better off).

The big problem with this procedure is that the answer you get depends on which basket you choose to track. Look again at Exhibit 3.12. Basket *O* costs $20 under the original prices and $20 under the new prices. If you track basket *O,* you'll say the cost of living hasn't changed at all. That's misleading, because, as we've just seen, Harold is definitely happier with the new prices than with the old ones.

If you tracked basket *P,* you'd get a different answer. Basket *P* is outside the *Original* budget line, which means it must cost more than $20 at the original prices. But it is

exactly on the *New* budget line, meaning it costs just $20 at the new prices. So basket *P* does get cheaper over time, and if you used it to measure the cost of living you'd say that the cost of living had come down.

The cost of living measurement that you get by tracking the original basket (in this case *O*) is called a **Laspeyres price index** (pronounced "La-spears"), and it tends to make things look worse than they are. The cost of living measurement that you get by tracking the new basket (in this case *P*) is called a **Paasche price index** (pronounced "Posh"), and it tends to make things look better than they are. Unfortunately, there is no perfect way to measure changes in the cost of living.

Differences in Tastes

Germans eat a lot of starch. Italians eat more tomatoes. Greeks use olive oil and the French use hollandaise. Why doesn't everyone eat the same diet?

There are only two possible answers: People in different countries must have either different tastes or different opportunities (or both). Maybe Italians eat tomatoes because they like them better than Germans do—that's a difference in tastes. Or maybe Italians eat tomatoes because tomatoes are cheaper in Italy, or because Italians are too poor to afford a German diet—those are differences in opportunities.

How do we tell which theory is right? There's no question that prices and incomes differ across countries, so there's no question that there are differences in opportunities. The question is whether those differences in opportunities suffice to explain the different choices people make, or whether their tastes must also differ.

Start with a fictional example: Suppose Albert lives in Rome, where tomatoes sell for $2 a pound and potatoes sell for $1 a pound. He earns $10 a day, with which he buys 4 tomatoes and 2 potatoes. Betty lives in Berlin, where tomatoes sell for $3 a pound and potatoes sell for $6 a pound. She earns $45 a day, with which she buys 1 tomato and 7 potatoes. Using these numbers, let's figure out whether Albert and Betty could have identical tastes.

The first step is to plot Albert and Betty's budget lines, which we've done in panel A of Exhibit 3.13. Albert's optimum point (4, 2) is labeled *A*, and Betty's optimum point (1, 7) is labeled *B*.

Exercise 3.8 Make sure the budget lines are drawn correctly.

Now let's change the problem slightly. Suppose that instead of buying 1 tomato and 7 potatoes, Betty buys 5 tomatoes and 5 potatoes. Then the picture looks like panel B in Exhibit 3.13. Here we can't tell whether the indifference curves eventually cross, and we can't tell whether Albert and Betty have identical tastes.


Dangerous Curve

To conclude that Albert and Betty have identical tastes, we would have to know that they share all their indifference curves. There are several reasons why we can't draw this conclusion from Exhibit 3.13B. First, we have no idea whether the two pictured indifference curves eventually cross. Second, even if they don't cross, it doesn't follow that Albert and Betty share these indifference curves; it only follows that they *might*. Third, even if Albert and Betty share the two pictured indifference curves, it doesn't follow that they share *all* their indifference curves. So the picture does not contain nearly enough information to answer the question of whether Albert and Betty's tastes are identical.

EXHIBIT 3.13   **Comparing Preferences**

A. One possibility

B. Another possibility
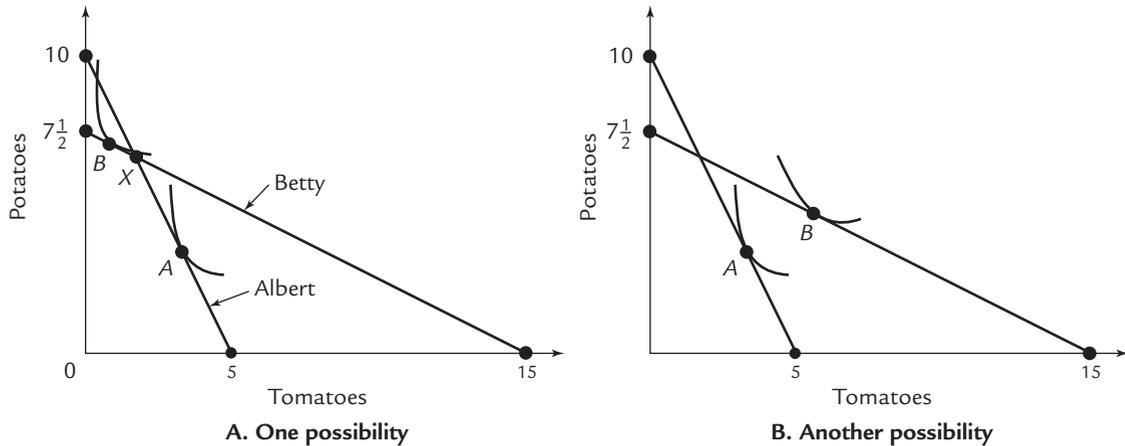
In panel A, Albert's indifference curve (tangent at *A*) must eventually cross Betty's indifference curve (tangent at *B*). Therefore, Albert and Betty cannot possibly have the same tastes. In panel B, the indifference curves might or might not cross and Albert and Betty might or might not have identical tastes.

Now that we've completed our detour into fiction, what about the real world? To seek evidence of taste differences across European countries, we can replace Albert and Betty with "the average German" and "the average Italian," and we can use realistic numbers for the prices of tomatoes and potatoes. Then we can repeat the exercise with Germans and Greeks, or Greeks and Italians, or Poles and Hungarians, and with more than just two goods. If we ever get a picture like panel A of Exhibit 3.13, we've spotted a taste difference.

Harvard Professor Hendrik Houthakker carried out this exercise and found no evidence of any taste differences. In other words, when Professor Houthakker drew his graphs, none looked like panel A of Exhibit 3.13. Instead, every one of his pictures leaves open the possibility that tastes could be either the same or different.

On the one hand, that doesn't prove anything. On the other hand, the more times you look for something and fail to find it, the more you're entitled to suspect it's not really there. Professor Houthakker searched repeatedly for evidence of taste differences and failed to find them. That doesn't prove tastes are remarkably similar across countries, but it is certainly evidence in that direction.

Here's a similar question: Do people's tastes change over time? For example, did the average Englishman in 1950 have different tastes than the average Englishman in 1900? We can use the same techniques: In Exhibit 3.13, replace Albert and Betty with "the average Englishman in the year 1950" and "the average Englishman in the year 1900." Look at not just tomatoes and potatoes but other pairs of goods. A picture like panel A of Exhibit 3.13 would show that tastes had changed over that half-century.

Using 127 different goods in every possible pairing, there are many hundreds of cases where the budget lines cross, raising the possibility of a configuration like panel A

of Exhibit 3.13. In no case does that configuration actually occur.[3] In other words, there are a lot of opportunities to observe a taste change and no actual observations. Again, that doesn't prove anything, but it is highly suggestive.

The Least Bad Tax

Governments raise money in many ways.

First, there are *wage taxes* (also called *payroll taxes*). In the United States, the most important wage tax is the FICA tax that is used to fund the Social Security and Medicare programs. FICA taxes are deducted directly from your paycheck. In most cases, the deduction is about 7.5% of your pay. (Your employer pays a comparable amount.)

Next, there are *income taxes* that tax income from all sources—not just wages, but also income on interest, dividends, and so forth.

Then there are *consumption taxes* that tax the purchases you make. In the United States, most state governments gather large fractions of their income from consumption taxes (more commonly called sales taxes).

Local governments frequently raise funds through *property taxes,* where you pay a percentage of the value of your property, or more frequently, of certain kinds of property such as real estate.

A rarer form of tax is the *head tax* which requires you to pay a certain number of dollars per year, independent of your income or your consumption.

If you've got to be taxed, which is the least painful tax to pay?

Obviously, the answer depends on the size of the taxes. A 1% income tax is better than a $10,000 daily head tax, whereas a $1 daily head tax is better than a 90% income tax.

So to keep the comparison fair, let's assume in each case that you're going to pay exactly, say, $100 a year in taxes. Now which tax is least painful?

A reasonable guess is: If two taxes each cost me $100 a year, then those two taxes are equally painful. This reasonable guess turns out to be wrong, because different taxes lead to different changes in behavior, and some of these changes are less desirable than others.

It's easiest to see this with an extreme example. Which would you prefer—a 0% income tax or a 5,000,000% tax on shoes? Under the 0% income tax, you pay zero. Under the 5,000,000% shoe tax, you also pay zero (because you decide to go barefoot). Either tax costs you the same zero dollars, but the shoe tax is worse because it leaves you with cold feet.

So even when two taxes collect exactly the same amount of revenue, we can't assume they're equally painful. We have to consider their effects on your behavior.
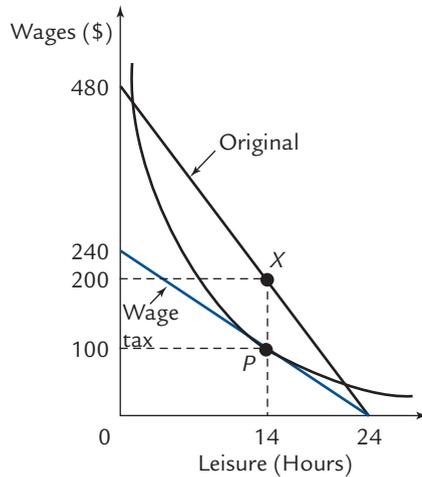
Wage Taxes versus Head Taxes

For example, when wages are taxed, people might choose to work less and earn fewer wages. To understand the consequences of this choice, we need an indifference curve diagram with "wages earned" on one axis and . . . what on the other? Answer: The alternative to earning wages is having more leisure, so that's what goes on the other axis.

We assume your wage rate is $20 an hour. You can divide your 24-hour day any way you want between leisure and wage-earning. If you take zero hours of leisure (working a full 24 hours) you earn $480 in wages; if you take 24 hours of leisure you earn $0 in wages. These are the endpoints of the budget line labeled *Original* in panel A of Exhibit 3.14. Deciding how many hours to work amounts to choosing a point on this budget line.
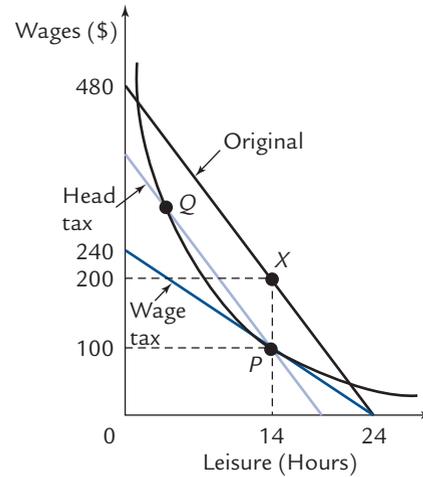
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[3] S. Landsburg, "Taste Change in the United Kingdom," *Journal of Political Economy* 89 (1981): 92–104.

EXHIBIT **3.14**   **A Wage Tax versus a Head Tax**

A. The effect of a wage tax

B. A wage tax versus a head tax

You have 24 hours a day to divide between leisure and working at a wage of $20 an hour; this yields the *Original* budget line. When your wages are taxed at 50%, the budget line pivots to the *Wage Tax* line in panel A, and you choose point *P.*

If the wage tax is replaced by a head tax that collects the same number of dollars, the new budget line must be parallel to the *Original* (because it represents a head tax) and must pass through point *P* (because it raises the same number of dollars as the wage tax). This yields the *Head Tax* line in panel B, which enables you to reach a higher indifference curve (with a tangency somewhere between *P* and *Q*). Thus the head tax is preferable to the wage tax.

Now suppose you are subject to a wage tax that lowers your after-tax wage from $20 an hour to $10 an hour. Then if you take zero hours of leisure, you earn only $240 in wages. In other words, your budget line pivots inward to the line labeled *Wage Tax* in panel A of Exhibit 3.14.

With the wage tax in effect, you must choose a point on this *Wage Tax* line. Of course you choose the point where an indifference curve is tangent; in the exhibit we have labeled this point *P.* For illustration, we've assumed that *P* has coordinates (14, 100). In other words, you spend 14 hours at leisure and 10 hours at work.

In the absence of a tax, 10 hours of work would have earned you $200, so this wage tax is taking $100 out of your paycheck. To put this another way, if you had worked 10 hours *without* being taxed, you'd have been at point *X,* $100 directly above point *P.*

Sometimes students think that *X* must be the optimum on the *Original* budget line. There is no reason to expect this. In the absence of a wage tax, you'd probably work some number of hours other than 10, which is to say that the optimum on the *Original* line is probably somewhere other than *X.* What we're saying here is that if there were no wage tax *and if for some reason you still worked exactly 10 hours,* then you'd be at point *X.*

Dangerous
Curve

Now let's compare the wage tax to a head tax. Remember that we want to keep the comparison fair by assuming that each tax collects the same number of dollars. The wage tax collected $100, so we have to compare it to a $100 head tax.

A head tax takes $100 from you no matter how much you work; therefore the entire *Original* budget line is shifted vertically downward (parallel to itself) a distance of $100. Because the *Original* line goes through point *X*, the shifted *Head Tax* line must go through point *P*, exactly $100 below *X*, as shown in panel B of Exhibit 3.14.

With the head tax in effect, you choose a point of tangency on the *Head Tax* line. To avoid crossings, this tangency must lie somewhere between points *P* and *Q*. Notice that any such tangency must lie on a *higher* indifference curve than the one pictured. Therefore:

A head tax is preferable to a wage tax (assuming that both taxes collect the same number of dollars).

What's going on here? How can one tax be preferable to another when they're equally costly? Answer: Suppose you're subject to the wage tax, so your after-tax wage is $10 per hour. Then you choose point *P* in panel A of Exhibit 3.14; at this point the marginal value of your leisure (measured by the slope of your indifference curve) is exactly $10 per hour. Now if the wage tax is suddenly lifted and replaced with a head tax, you have the opportunity to earn $20 for each additional hour of work. Since you value your next hour of leisure at only $10 an hour, you'll be glad to be able to sell it for $20.

Wage Taxes versus Income Taxes

The difference between a wage tax and an income tax is that while a wage tax discourages work, an income tax discourages both work and saving. (It discourages saving by taxing interest, which is the reward for saving.) To compare the two taxes, then, we need an indifference curve diagram with saving on one of the axes. Alternatively, we can put "future consumption" on one of the axes, because the entire purpose of saving is to enable future consumption.

The alternative to saving is current consumption, so that's what goes on the other axis; you can see the picture in panel A of Exhibit 3.15. Here we suppose that you've just earned $100 in wages. You can spend part of your $100 today and save the rest for tomorrow—say at a 10% interest rate. If you spend everything now, you get $100 worth of current consumption and $0 worth of future consumption; if you save everything, you get $0 worth of current consumption and $110 worth of future consumption. The *Original* budget line shows your menu of choices.

Now consider the effect of a tax on interest only. (We'll return to income taxes shortly, but it's important to understand the effects of interest-only taxes first.) If, say, half your interest earnings are taxed, then you earn an after-tax interest rate of only 5%. You can still spend your entire $100 today, but if you save it all for the future you'll have only $105 to spend. So your budget line pivots as in panel A of Exhibit 3.15.

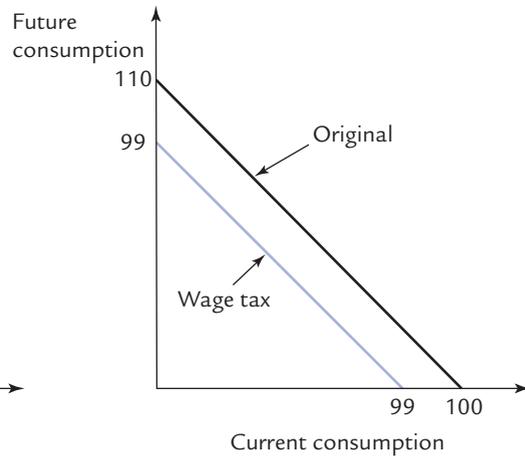A tax on interest causes the budget line to pivot.

For comparison, panel B shows the effect of a wage tax, say of 10%. Your after-tax wage is reduced to $90; you can spend the entire $90 today, save it all and spend $99 tomorrow, or do anything in between, as the *Wage Tax* line indicates. Note that the slope of the budget line is unchanged; a wage tax causes this budget line to shift *parallel to itself.*
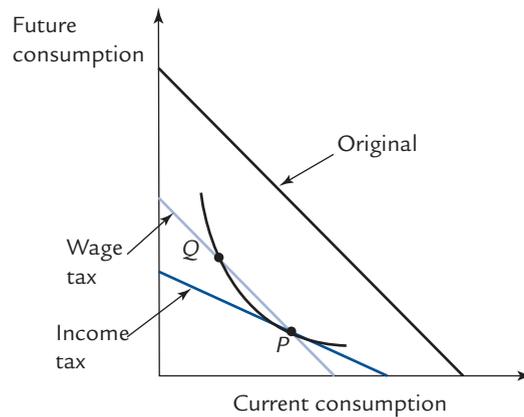
EXHIBIT **3.15** **Taxing Interest, Wages, and Income**
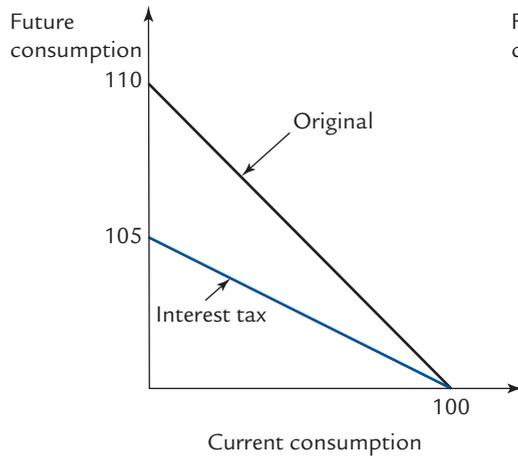
A. A tax on interest

B. A tax on wages

C. An income tax versus a wage tax

You have $100 to divide between current consumption and saving for future consumption; your savings earn a 10% interest rate. This gives you the *Original* budget line.

A tax on interest causes the budget line to pivot (panel A); a tax on wages causes the budget line to shift in parallel to itself (panel B). A tax on income (i.e., on both interest and wages) causes both a pivot and a parallel shift, yielding the *Income Tax* line in panel C.

If the income tax is replaced with a wage tax that collects the same number of dollars, the new *Wage Tax* line is parallel to the *Original* and passes through point *P*. This allows you to reach a higher indifference curve (with a tangency somewhere between *P* and *Q*). Thus the wage tax is preferred to the income tax.

In Exhibit 3.14, a wage tax causes the budget line to *pivot,* whereas in Exhibit 3.15, a wage tax causes the budget line to make a *parallel shift.* This is because neither diagram by itself shows the full effect of a wage tax. Ideally, we'd have a four-dimensional diagram with axes for "Current Consumption," "Future Consumption," "Current Leisure," and "Future Leisure." Our paltry two-dimensional diagrams show only pieces of the story. The wage tax causes a pivot in one dimension and a parallel shift in another.

Dangerous Curve

Panels A and B of Exhibit 3.15 show the effects of an *interest tax* and a *wage tax*. Now what about an *income tax?* An income tax is a tax on both interest *and* wages, so it is represented by both a pivot *and* a shift of the budget line. You can see the *Income Tax* line in panel C. Notice that because of the pivot, it is *not* parallel to the *Original* line.

When you are subject to the income tax, you choose point *P.* Now what if the income tax is replaced by a wage tax that collects the same number of dollars? The *Wage Tax* line must be parallel to the *Original* line (because it represents a wage tax) and must pass through point *P* (because it collects the same number of dollars as the income tax).

Dangerous Curve

When we're talking about multiple time periods, we have to be careful about what it means for two taxes to collect "the same number of dollars." Given our assumption of a 10% interest rate, we have to count a dollar collected today as equivalent to $1.10 collected tomorrow.

As you can see in panel C, the wage tax allows you to reach a tangency between *P* and *Q,* at a point better than *P.* In other words:

A wage tax is preferable to an income tax (assuming that both taxes collect the same number of dollars).

You should note that Exhibit 3.15C is very similar to Exhibit 3.14B. In each case, the moral is that **a tax that causes the budget line to pivot is worse than a tax that causes a pure parallel shift.**

Consumption Taxes

Finally, what about a consumption tax? For this, we can look again at Exhibit 3.15B. The picture assumes you have $100, which you can divide between current consumption and saving for future consumption, at a 10% interest rate.

Now suppose you have to pay a sales tax, so that your $100 can only purchase $90 worth of goods. That means you can have up to $90 worth of current consumption. Alternatively, if you save all your money, you'll have $110, which, at the same tax rate, will allow you to buy and consume up to $99 worth of future goods. In other words, your new budget line passes through the points (0, 90) and (99, 0)—*just like the illustrated* Wage Tax *line!* In still other words, the *Consumption Tax* line is *identical* to the *Wage Tax* line.

So a consumption tax and a wage tax (of appropriate sizes) cause your budget line to shift in exactly the same way. Since both taxes yield the same budget line, they must also yield the same optimum—which means there's no reason for you to prefer one over the other.

A wage tax and a consumption tax are interchangeable; there is no reason to prefer one over the other.

Conclusion

The head tax is best, the wage tax is second best, the consumption tax ties with the wage tax, and the income tax comes in last.

Once again, it's important to remember that all this assumes you're paying the same amount under any of the various tax systems. If you are the average taxpayer, this can be a reasonable assumption; as long as the government sets the tax rate with a target revenue in mind, it will presumably adjust that tax rate to keep total collections fixed.

But not every taxpayer is average. If your income is considerably below average, you might prefer an income tax to a head tax; if your interest income is considerably below average, you might prefer an income tax to a wage tax or a consumption tax.

Summary

A consumer's behavior depends on his tastes and his opportunities. His tastes are encoded in his indifference curves and his opportunities are encoded in his budget line. By combining this information in a single graph, we can predict the consumer's behavior.

Each consumer has a family of indifference curves. Each curve in the family consists of baskets among which he is indifferent. His indifference curves slope downward, fill the plane, never cross, and are convex. A different consumer will have a different family of indifference curves, also satisfying these properties.

The slope of an indifference curve is equal (in absolute value) to the marginal value of X in terms of Y. That is, it is the number of units of Y for which the consumer is just willing to trade one unit of X.

As the consumer moves along an indifference curve in the direction of more X and less Y, we expect that the marginal value of X will decrease. This accounts for the convexity of indifference curves.

The consumer's budget line depends on his income and the prices of the goods that he buys. Its equation is

$$P_X \bullet x + P_Y \bullet y = I$$

where P_X and P_Y are the prices of X and Y and I is the consumer's income. The slope of the budget line is equal (in absolute value) to the relative price of X in terms of Y.

The consumer's optimum occurs where his budget line is tangent to one of his indifference curves. This is the point at which he attains the highest indifference curve that is available to him. At this point the marginal value of X in terms of Y is equal to the relative price of X in terms of Y. At any other point either the marginal value would exceed the relative price, in which case the consumer would trade Y for X, or the relative price would exceed the marginal value, in which case the consumer would trade X for Y. Only at his optimum point is he satisfied not to trade any further.

Author Commentary   www.cengage.com/economics/landsburg

AC1. See these articles for some challenges to the budget line/indifference curve model of consumer choices.

Review Questions

R1. Consider the baskets $A = (3, 4)$, $B = (5, 7)$, $C = (4, 2)$. Without knowing Beth's indifference curves, can you predict which of these baskets she'll like the best? Can you predict which she'll like the least?

R2. Explain why indifference curves must slope downward.

R3. Explain why two of Beth's indifference curves can never cross.

R4. Can one of Beth's indifference curves cross one of Carol's indifference curves? Why or why not?

R5. Define the "marginal value of X in terms of Y."

R6. Suppose Beth owns basket (10, 10) and the slope of her indifference curve at that point is 4. Would Beth be willing to trade her basket for the basket (9, 13)?

R7. Write the equation for a consumer's budget line. Which symbols represent constants and which represent variables?

R8. Susan has an income of $10. She buys cherries for $2 a pound and grapes for $4 a pound. Write the equation for her budget line and sketch the line. What is its slope?

R9. Given the consumer's indifference curves and budget line, how do you find the consumer's optimum point?

R10. Suppose the marginal value of X in terms of Y is greater than the relative price of X in terms of Y. Is the consumer's basket to the left or to the right of his optimum point? Will he want to buy some X or to sell some X? Explain how you know. In which direction will this cause the consumer to move along his budget line?

Numerical Exercises

N1. Every day Fred buys wax lips and candy cigarettes. After deciding how many of each to buy, he multiplies the number of sets of wax lips times the number of packs of candy cigarettes. The higher this number comes out to be, the happier he is. For example, 3 sets of wax lips and 5 packs of candy cigarettes will make him happier than 2 sets of wax lips and 7 packs of candy cigarettes, because 3×5 is greater than 2×7. Wax lips sell for $2 a pair and candy cigarettes for $1 a pack. Fred has $20 to spend each day.

  a. Make a table that looks like this:

Pairs of Wax Lips	Packs of Candy Cigarettes
0	
1	
2	
.	
.	
.	
10	

  where each row of the chart corresponds to a basket on Fred's budget line. Fill in the second column.

b. Draw a graph showing Fred's budget line and marking the baskets described by your table. Draw Fred's indifference curves through these baskets. If he must select among these baskets, which one will Fred choose?

c. Add to your table a third column labeled *MV* for the marginal value of wax lips in terms of candy cigarettes. Fill in the *MV* for each basket. (*Hint:* For each basket construct another basket that has one less pair of wax lips but enough more packs of candy cigarettes to be equally desirable. How many packs of candy cigarettes have been added to the basket?) For which basket is the marginal value closest to the relative price of wax lips? Is this consistent with your answer to part (b)?

Problem Set

1. **True or False:** If the price of wine rises, peoples' tastes will shift away from wine and toward other things.

2. Suppose that you like to own both left and right shoes, but that a right shoe is of no use to you unless you own a matching left one, and vice versa. Draw your indifference curves between left and right shoes.

3. Draw your indifference curves between nickels and dimes, assuming that you are always willing to trade 2 nickels for 1 dime, or vice versa. What is the marginal value of nickels in terms of dimes?

4. Judith loves cats, hates dogs, and is completely indifferent to tropical fish. Draw her indifference curves between (a) cats and dogs, (b) cats and fish, (c) dogs and fish.

5. Suppose that you hate typing and hate filing.

a. Draw a graph with "hours of typing" on the horizontal axis and "hours of filing" on the vertical. Do your indifference curves slope upward or downward? Why?

b. Suppose you currently type for 3 hours a day and file for 5, but you'd be just as happy typing for 2 hours a day and filing for 7. What is the slope of your indifference curve at the point (3, 5)? If you hated typing even more than you do, would you expect the indifference curve to be steeper or shallower?

c. Would you expect the indifference curve to be steeper or shallower at points that represent a lot of typing and very little filing? What does this say about the shape of the indifference curves?

d. Suppose your boss tells you that henceforth, you may divide your 8-hour day any way you wish between these two activities, but the number of hours you spend typing and the number of hours you spend filing must add up to 8. Draw the relevant budget constraint.

e. Given the information in part (b), will you now choose to type more or less than 3 hours a day? Illustrate your new optimum and explain why it is your optimum.

6. Filbert is indifferent between baskets (3, 2) and (4, 1). Lychee is indifferent between baskets (1, 4) and (2, 3). Note that all four baskets lie along a straight line.

   a. Can you determine whether Filbert and Lychee have identical tastes?

   b. Suppose that Filbert chooses basket (4, 1) and Lychee chooses basket (1, 4). Can you determine whether Filbert and Lychee pay identical prices for the goods they buy?

7. Huey consumes only two goods, X and Y. His indifference curves have the usual shape. He prefers basket (1, 3) to basket (2, 2).

   a. Is it possible to tell whether Huey prefers (1, 3) to (3, 1)?

   b. Is it possible to tell whether Huey prefers (3, 1) to (2, 2)?

8. Suppose your indifference curves between food and clothing were nonconvex as in Exhibit 3.10. **True or False:** In this case a very small change in price could lead to either no change at all in your consumption of X or to a very large change in your consumption of X.

9. Suppose that you consume nothing but beer and pizza. In 2010, your income is $10 per week, beer costs $1 per bottle, pizza costs $1 per slice, and you buy 6 bottles of beer and 4 slices of pizza per week. In 2011, your income rises to $20 per week, the price of beer rises to $2.50 per bottle, and the price of pizza rises to $1.25 per slice.

   a. In which year are you happier?

   b. In which year do you eat more pizza? Justify and illustrate your answer with indifference curves.

10. In 2010, you buy shoes for $2 a pair and socks for $1 a pair, and your income is $30, with which you buy 12 pairs of shoes and 6 pairs of socks. In 2011, you buy shoes for $1 a pair and socks for $2 a pair, and your income is still $30.

   a. Draw both years' budget lines. Notice that they cross at the point (10, 10).

   b. **True or False:** In 2011, you will surely buy more than 10 pairs of shoes.

   c. **True or False:** In 2011, you will surely buy more than 12 pairs of shoes.

11. Your income is $48, which you spend on eggs and wine. Eggs sell for $4 a dozen. Every day you buy 5 dozen eggs. One day the egg salesman offers you a deal: "If you pay $10 a day to join the egg club, you'll be allowed to buy eggs at $2 a dozen." Should you join the club? Justify your answer with indifference curves.

12. If the price of eggs were to double from $1 per egg to $2 per egg, Freddy would consume 6 fewer eggs without changing his consumption of other goods. Which would he prefer: The price increase, or losing $6?

13. Aubrey buys only apples and peaches. In June, apples sell for $2 each and peaches sell for $1 each. In July, apples sell for $1 each and peaches sell for $2 each. Aubrey's income is $20 in June and $20 in July.

   a. **True or False:** If Aubrey is equally happy in both months, then she surely eats more apples in July.

   b. **True or False:** If Aubrey buys exactly eight apples in June, then she is certainly happier in July.

   c. **True or False:** If Aubrey buys exactly eight apples in June, then she certainly buys more than eight apples in July.

14. Cassie shops at Wegman's supermarket, where she spends $20 a week to buy 10 apples and 5 bananas. If she bought the same 10 apples and 5 bananas at Top's supermarket, she'd pay $30. **True or False:** Cassie is wise to continue shopping at Wegman's.

15. Gregorian and Boudicca each have incomes of $30 and each shop at Star Market, where apples cost $2 each and bananas cost $1 each. Every day, Gregorian buys 6 apples and 18 bananas, and so does Boudicca. One day a new supermarket (called Acme) opens up. At Acme, apples cost $1 and bananas cost $2. Gregorian prefers to keep shopping at Star Market, but Boudicca switches to Acme.

    a. Draw a diagram showing Gregorian's budget lines at Star Market and Acme. Illustrate his indifference curves. Do the same for Boudicca.

    b. What is the key difference between the shapes of Gregorian's and Boudicca's indifference curves?

    c. **True or False:** At Acme, Boudicca will surely buy more than 10 apples.

    d. The president of Star Market Points out that at Acme, Boudicca will no longer be able to afford the basket she's been buying at Star Market. Is this a good reason for Boudicca to reconsider her choice?

16. Amelia buys coffee for $1 per cup and tea for 50¢ per cup; every day she drinks 1 cup of coffee and 2 cups of tea. Bernard buys coffee for 50¢ per cup and tea for $1 per cup; every day he drinks 2 cups of coffee and 1 cup of tea. Can you determine whether Amelia and Bernard have identical tastes?

17. Chris buys coffee for $1 per cup and tea for 50¢ per cup; every day she drinks 2 cups of coffee and 1 cup of tea. David buys coffee for 50¢ per cup and tea for $1 per cup; every day he drinks 1 cup of coffee and 2 cups of tea. Can you determine whether Chris and David have identical tastes?

18. Evelyn buys coffee for $1 per cup and tea for 50¢ per cup; every day she drinks 1 cup of coffee and 2 cups of tea. Frederick buys coffee for 50¢ per cup and tea for $1 per cup; every day he drinks 1 cup of coffee and 1 cup of tea. Can you determine whether Evelyn and Frederick have identical tastes?

19. John buys eggs for $2 a dozen and bacon for $5 a pound. Sarah buys eggs for $5 a dozen and bacon for $2 a pound. Can you determine whether John and Sarah have identical tastes?

20. In each of the three circumstances (a, b, and c below), determine which of the following conclusions (1, 2, or 3) holds and justify your answer:

    (1) John and Mary have identical tastes.

    (2) John and Mary have different tastes.

    (3) We can't tell whether John and Mary have identical tastes.

    a. John and Mary have the same budget line and choose different baskets.

    b. John and Mary have the same budget line and choose the same basket.

    c. John and Mary have crossing budget lines and choose the same basket.

21. John buys shoes for $1 a pair and socks for $1 a pair. His annual income is $20.

    a. Draw John's budget line.

    b. Now suppose the government institutes two new programs: First, it taxes shoes, so that shoes now cost John $2 a pair. Second, it gives John an annual cash gift of $10. Draw his new budget line.

    c. Suppose that with the new programs in place, John chooses to buy 10 pairs of socks and 10 pairs of shoes. Has the pair of government programs made him better off, worse off, or neither?

22. a. Suppose you have 16 waking hours per day, which you can allocate between leisure and working for a wage of $10 an hour. Draw your budget constraint between "leisure" (measured in hours) and "income" (measured in dollars).

b. Suppose you invent a pill that enables you to get by on four hours of sleep a night, so that you now have 20 waking hours per day. Is it possible that you will now choose to work fewer hours than before?

23. The Pullman company has a lot of pull in the town of Pullman. Everybody in town is identical, and they all work for the company, which pays them each $10 a day. Their favorite food is apples, which they get from a mail-order catalog for $1 apiece.

a. Draw the typical resident's budget line between apples and all other goods, with all other goods measured in dollars.

b. Pullman has decided to institute a sales tax of $1 per apple. But to prevent dissatisfied workers from leaving town, Pullman must simultaneously raise wages so that workers are just as happy as before. Draw the typical resident's new budget line, given both the sales tax and the wage increase.

c. Use your graph to illustrate Pullman's new net expense per worker (that is, wages paid minus sales tax collected).

d. Was Pullman wise to institute the tax?

24. Suppose that you can work anywhere from 0 to 24 hours per day at a wage of $1 per hour. You are subject to a tax of 50% on all wages over $5 per day (the first $5 per day is untaxed). You elect to work 10 hours per day.

a. Show your budget constraint between labor and wages, and show your optimum point.

b. Suppose that the tax law is changed so that *all* wages are subject to a 25% tax. Do you now work more or less than 10 hours? Does the government collect more or less tax revenue than before?

c. Which do you prefer: the old tax law or the new one?

25. Suppose that you have 24 hours per day to allocate between leisure and working at a wage of $1 per hour. Draw your budget line between leisure and dollars. One day the government simultaneously institutes two new programs: a 50% income tax and a plan whereby everybody in the country receives a gift from the government of $6 each year.

a. Draw your new budget line.

b. Suppose that the government chose the level of $6 for the gift because it precisely exhausts the income from the tax. Explain why this means that the average taxpayer must be paying exactly $6 in tax.

c. Assume that you are the average taxpayer, and draw your new optimum. Is it on, above, or below your original budget line?

d. As the average taxpayer, are you working harder or less hard than before the programs went into effect? Are you happier or less happy? How do you know?

26. Suppose the government imposes a *temporary* sales tax—one that is in effect for a short time, but will disappear in the future. The government is considering two different tax policies:

A. A big excise tax on eggs. This would cause the price of eggs to triple.

B. A smaller excise tax on both eggs and wine. This would cause the price of both eggs and wine to double.

a. Illustrate your original (no-tax) budget line and your budget line under Policy *A.* Mark your optimum point.

b. Suppose that, coincidentally, the government would collect exactly as much money from you under Policy *B* as under Policy *A.* Illustrate your budget line under Policy *B.* How does your graph illustrate the fact that the two policies cost you equal amounts of money?

c. Which policy do you prefer? Why?

27. Suppose the government imposes a *temporary* sales tax—one that is in effect for a short time, but will disappear in the future. In a diagram relating current consumption to future consumption, how does your budget line shift? Which is preferable: a permanent sales tax or a temporary sales tax (assuming the rates are adjusted so they collect equivalent revenues)?

Appendix to Chapter 3

Cardinal Utility

The theory of cardinal utility is an alternative approach to consumer behavior. It has the advantage of sometimes being easier to work with and the disadvantage that it introduces a new quantity—called *utility*—that can never actually be measured. However, it turns out to be the case that *the cardinal utility approach has exactly the same implications as the indifference curve approach.* Thus, the choice between the two is largely a matter of convenience and of taste.

The Utility Function

In the cardinal utility approach, we assume that the consumer can associate each basket with a number, called the **utility** derived from that basket, that measures how much pleasure or satisfaction he would get from owning that basket. For the basket containing x units of X and y units of Y, the utility is often denoted $U(x, y)$. Thus, for example, if we write

$$U(5, 7) = 6$$

what we mean is that a basket containing 5 Xs and 7 Ys gives the consumer 6 units of utility. The rule for going from baskets to utilities is called the consumer's *utility function.* An example of a utility function is

$$U(x, y) = \sqrt{xy + 1}$$

which would yield the value $U(5, 7) = 6$, as above.

We assume that, given a choice between two baskets, the consumer always chooses the one that yields higher utility. Thus, if the consumer with the preceding utility function were given a choice between basket *A,* with 5 units of X and 7 units of Y, and basket *B,* with 6 units of X and 4 units of Y, then he would choose basket *A,* because $U(5, 7) = 6$ but $U(6, 4) = 5$.

The assumption that consumers seek to maximize utility enables us to pass from utility functions to indifference curves. The consumer with this utility function is indifferent between the baskets (6, 4), (8, 3), (12, 2), and (4, 6), because they all yield utilities of 5. Thus, all of these baskets must lie on the same indifference curve. More generally, all of the baskets (x, y) that satisfy

$$U(x, y) = 5$$

lie on a single indifference curve, so that the equation of that indifference curve is given by $U(x, y) = 5$. Similarly, there is another indifference curve whose equation is given by $U(x, y) = 6$.

Utility

A measure of pleasure or satisfaction.

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If a consumer has the utility function $U(x, y)$, then his indifference curves are the curves with equations $U(x, y) = c$, where c is any constant.

Marginal Utility

The consumer's **marginal utility of X (MU_X)** is defined to be the amount of additional utility he acquires when the amount of X is increased by one unit and the amount of Y is held constant. For example, consider a consumer whose utility function is as given and who consumes 5 units of X and 7 units of Y. His utility is $U(5, 7) = 6$. If we increase his consumption of X by one unit, his utility will be $U(6, 7) \approx 6.557$. Thus, the marginal utility of X for this consumer is about .557.

We define the marginal utility of Y (MU_Y) in a similar way. For this consumer, increasing Y by one unit would yield utility $U(5, 8) \approx 6.403$. The marginal utility of Y for this consumer is about .403.

We assume that the marginal utility of X is always positive (more is preferred to less) but that each additional unit of X yields less marginal utility than the previous unit (always holding fixed the consumption of Y). This is known as the principle of *diminishing marginal utility*. For example, we have seen that a consumer who starts with basket (5, 7) has $MU_X \approx .557$. After acquiring a unit of X and moving to basket (6, 7), his marginal utility of X is reduced to $MU_X \approx .514$, as you can verify with your calculator.

Marginal Utility versus Marginal Value

We can relate the concept of marginal utility to the concept of marginal value. Suppose that we reduce your consumption of X by one unit. This reduces your utility by the amount MU_X. Now suppose that we increase your consumption of Y by ΔY units. This increases your utility by $MU_Y \bullet \Delta Y$. Finally, suppose that ΔY is chosen to leave you just as happy as you were before the changes in your consumption. Then ΔY is the marginal value (to you) of X in terms of Y forgone. Because you are equally happy before and after the changes, the loss of utility from consuming less X must equal the gain in utility from consuming more Y; in other words,

$$MU_X = MU_Y \bullet \Delta Y$$

Rearranging terms, we get

$$\frac{MU_X}{MU_Y} \bullet \Delta Y = MV_{XY}$$

where MV_{XY} denotes the marginal value of X in terms of Y.

The Marginal Utility of Income

Suppose that a consumer facing prices P_X and P_Y finds that his income goes up by a dollar. How much additional utility can he achieve?

First, suppose that he spends the additional dollar entirely on X. Then he can purchase $1/P_X$ units of X, each of which yields an additional MU_X units of utility. By spending an additional dollar on X, the consumer increases his utility by the amount $MU_X \bullet (1/P_X) = MU_X/P_X$. Similarly, by spending an additional dollar on Y, the consumer increases his utility by the amount MU_Y/P_Y. We can think of MU_X/P_X and MU_Y/P_Y as the marginal utilities of a dollar spent on X and of a dollar spent on Y.

The Consumer's Optimum

The consumer allocates his income across X and Y so as to achieve the highest possible level of utility. We will determine the conditions that describe this optimum.

Consider the marginal utility of a dollar spent on X, MU_X/P_X, and the marginal utility of a dollar spent on Y, MU_Y/P_Y. We will argue that at the consumer's optimum these two quantities must be equal.

To see why, suppose first that MU_X/P_X, is greater than MU_Y/P_Y. Then there is a way for the consumer to increase his utility. He can spend one dollar less on Y and use that dollar to buy more of X. In doing so, he will sacrifice MU_Y/P_Y units of utility and gain the greater quantity MU_X/P_X; thus, he becomes better off. Having increased his consumption of X, the consumer finds, due to decreasing marginal utility, that MU_X is reduced; and having decreased his consumption of Y, he finds that MU_Y is increased. This brings the quantities MU_X/P_X and MU_Y/P_Y closer together. If MU_X/P_X still exceeds MU_Y/P_Y, the consumer will again cut his expenditures on Y and use the freed-up income to buy more of X. This continues until MU_X/P_X and MU_Y/P_Y become equal.

The same sort of thing happens if MU_Y/P_Y starts out greater than MU_X/P_X. In this case the consumer can increase his utility by spending less on X and more on Y, which brings MU_X/P_X and MU_Y/P_Y closer together. Again, the process continues until the two are equal.

Thus, at the consumer's optimum we must have

$$MU_X/P_X = MU_Y/P_Y$$

Rearranging terms, we get

$$MU_X/MU_Y = P_X/P_Y$$

We have encountered this last term before in this appendix; we determined that it is equal to the marginal value of X in terms of Y. The term on the right is the relative price of X in terms of Y. So our cardinal utility analysis leads us to conclude that the consumer's optimum occurs at that point on his budget line where the marginal value of X in terms of Y is equated to the relative price of X in terms of Y—exactly the same conclusion that we reached from the indifference curve analysis in Chapter 3!